Division of Engineering
BROWN UNIVERSITY
PROVIDENCE, R. I.

AFOSR TN-1594

EFFECTS OF CONDUCTION AND VISCOSITY ON THE STABILITY OF LAMINAR FLAME

BY

J.Y. PARLANGE and BOA-TEH CHU

Air Force Office of Scientific Research
Office of Aerospace Research
Contract No. AF 49(638)-646
Project No. 9751 Task No. 37510

AF 646/3

September 1961

AFOSR-1594

EFFECTS OF CONDUCTION AND VISCOSITY ON THE STABILITY OF LAMINAR FLAME

by

J.Y. Parlange and Boa-Teh Chu

Brown University

September, 1961

Air Force Office of Scientific Research

Contract No. AF 49(638)-646

Project No. 9751

Task No. 37510

Air Force Office of Scientific Research

Office of Aerospace Research

United States Air Force

F O R E W O R D

The present study is part of a program of "Theoretical Research in Combustion Aerodynamics" being conducted by the Division of Engineering, Brown University, under United States Air Force Contract No. AF 49(638)-646, Project No. 9751, Task No. 37510. The work was administered by the United States Air Force Office of Scientific Research.

SUMMARY

The effects of conduction and viscosity on the stability of laminar flame are examined. If $\mathcal{L}$ denotes the ratio of the wave length of a disturbance to the flame width and α is the ratio of the ultimate temperature of the burned gas to the initial temperature of the fresh mixture, the flame is found to be stable if

$$\mathcal{L} \leqslant \frac{2\alpha}{\alpha - 1} \frac{1}{R_e P_r}$$

where Re is the Reynolds number of the flame based on the flame width and Pr denotes the Prandtl number of the mixture. It is further shown that the stabilization is achieved primarily through the effect of heat concluction on the flame speed rather than the influence of viscosity.

Effects of Conduction and Viscosity on the Stability of Flame

1. Introduction We consider a mixture of two gases reacting with one another in accordance with the formula

$$-\eta_1 M_1 \rightleftarrows \eta_2 M_2 \tag{1.1}$$

Let m_α, Y_α, $Y_{o\alpha}$ be respectively the molecular weight of the species α, the mass-fraction of the species in the mixture and the mass-fraction of the species at upstream infinity. The composition of the mixture can be conveniently expressed by a variable ξ defined by

$$Y_\alpha = Y_{o\alpha} + m_\alpha \eta_\alpha \xi \tag{1.2}$$

We consider a low speed one-dimensional flow with a rapid change of temperature occurring in a narrow zone (width L) in which the chemical reaction (1.1) takes place. The low speed condition is expressed by saying that the Mach number of the flame satisfies

$$M \ll 1 \tag{1.3}$$

We shall study the stability of such a flame by considering a small perturbation. This allows us to linearize the problem. Let the x-axis be directed along the flow, y and z-axes parallel to the flame. We assume the solution is independent of z and that its dependence on y and time t are of the form: $e^{iky+\omega t}$

where k^{-1} is known and represents the wave length of the perturbation, while ω is unknown. If the real part of ω is positive, the flame is stable; if it is negative, it is unstable.

The flame profile is defined in terms of the temperature rather than the position coordinate x. It will be preferable to use T instead of x as one

of the independent variables inside the flame.

2. Flame Profile   Suppose that the enthalpy of the α species is given by

$$H_\alpha = C_p T + H_{o\alpha} \tag{2.1}$$

where Cp is the specific heat at constant pressure, assumed to be equal for both species, and $H_{o\alpha}$ denotes the enthalpy of formation. Let us define A by

$$A = m_1 \eta_1 [H_{o2} - H_{o1}] \tag{2.2}$$

If the Lewis number of the flame is equal to unity, the flame profile, expressed as a function of T, can easily be shown to be described by

$$\rho_o = \rho_{o\infty} \frac{T_{o\infty}}{T}$$

$$p_o = p_{o\infty}$$

$$u_o = u_{o\infty} \frac{T}{T_{o\infty}}$$

$$\xi_o = \frac{C_p}{A}(T - T_{o\infty})$$

when ρ , p , u denote respectively the density, pressure and velocity of the misture; the subscript "o" signifies that these quantities are associated with the unperturbed profile while subscript "∞" signifies the value of the quantity (to which it is appended) at upstream infinity. The flame profile is not completely defined until the position coordinate x is specified as a function of T. This last quantity, written as $x_o(T)$, depends on the production terms in the flame. Instead of specifying these terms, it is more convenient to assume a suitable form x such as that shown in Fig. 1. It is clear from this figure that the flame width L cannot be defined exactly. However, its order of magnitude is known.

3. Remarks on the Perturbed Flow   Consider a point $x_o(T)$ on the constant temperature surface T in the flame. When the flame is perturbed, this point

moves to the point $x_0(T) + x$. When the perturbation is small,

$$\frac{\partial x}{\partial T} \ll \frac{d x_0}{d T} \tag{3.1}$$

If $D(T)$ is the displacement of the constant temperature surface whose temperature is T, it is clear that

$$x \sim D, \quad x_0 \sim L \tag{3.2}$$

When the flame displacement is much less than the flame thickness,

$$D \ll L \tag*{(3.2)^1}$$

it is easy to see that D is independent of T. For, the flame thickness is related to the mean free path $\bar{I}$ by

$$L \sim \frac{\bar{I}}{M} \tag{3.3}$$

where M denotes the Mach number of the flame. Since the fluid dynamic equations apply only within the continuum limits, all length scales must be much greater than $\bar{I}$. In particular,

$$D \ggg \bar{I} \tag{3.4}$$

A comparison of (3.2) and (3.4) shows that D is independent of T so that inside the flame:*

$$\frac{\partial x}{\partial T} = 0 \tag{3.5}$$

It is possible to show that for

$$D \sim L \ (\text{or } D \gg L); \quad Pr \sim 1, \quad Re \sim 1 \tag{3.6}$$

where Pr denotes Prandtl number and Re denotes Reynolds number based on the flame thickness, viscosity and heat conduction effects are negligible and the flow is unstable. For the viscous effect to influence the flame stability, we

*If $D \sim L$ or greater, (3.1) shows that (3.5) is still true.

must have

$$\frac{Re}{kL} \sim 1 \tag{3.7}$$

and, for the conduction to affect the flame stability,

$$\frac{Re\,Pr}{k\,L} \sim 1 \tag{3.8}$$

In order that conduction and viscosity may affect the flame stability and a continuum analysis be meaningful, it is sufficient that

$$D \sim L \; ; \quad Re, Pr \ll 1 \; ; \quad kL \ll 1 \tag{3.9}$$

It is true the condition Re, Pr $\ll$ 1 is unrealistic. However, we shall show later that the more significant case encountered in practice and characterized by

$$D < L \; ; \quad kL < 1 \; ; \quad Re, Pr < 1 \tag{3.10}$$

can be considered as a limiting case of (3.9). It is, therefore, sufficient to focus our attention on (3.9) which is easier to handle from the mathematical viewpoint.

The condition, $x = D =$ a constant, valid inside the flame naturally indicates the interest of dividing the flow field into three regions:

$$\left.\begin{array}{lll} 1) & \text{Upstream:} & \dfrac{T - T_{0\infty}}{T_{0\infty}} \ll 1 \\[2ex] 2) & \text{Flame:} & \dfrac{T - T_{0\infty}}{T_{0\infty}} \sim 1 \\[2ex] 3) & \text{Downstream:} & \dfrac{T - \alpha T_{0\infty}}{\alpha T_{0\infty}} \ll 1 \end{array}\right\} \tag{3.11}$$

where α is the ratio of the temperature at downstream infinity to that at upstream infinity. Since the temperature profile is a continuous function of x, the definition (3.11) for the combustion zone is not well-defined. On the other hand, we may define the combustion zone by:

$$
\left.
\begin{array}{lll}
\text{1) Upstream:} & \dfrac{T-T_{0\infty}}{T_{0\infty}} \ll 1 \, , & \dfrac{dx_0}{dT} \gg \dfrac{L}{T_{0\infty}} \\[2ex]
\text{2) Flame:} & & \dfrac{dx_0}{dT} \sim \dfrac{L}{T_{0\infty}} \\[2ex]
\text{3) Downstream:} & \dfrac{T-\alpha T_{0\infty}}{\alpha T_{0\infty}} \ll 1 \, , & \dfrac{dx_0}{dT} \gg \dfrac{L}{T_{0\infty}}
\end{array}
\right\} \qquad (3.12)
$$

The new definition tolerates a discontinuity in derivatives. However, such a profile is not so well adapted for studying the effect of viscosity.

4. Basic Equations The basic equations governing the motion of a reactive mixture are well known. (See for example, ref. 1 or 2). These equations may be rewritten in terms of the new variables

$$
\begin{aligned}
T &= T(x, y, t) \\
\eta &= y \\
\tau &= t
\end{aligned}
\qquad (4.1)
$$

inside the flame. When the flame is slightly perturbed, the disturbances are governed by the linearized version of these equations. The nondimensional form of these linearized equations is given below. We shall use the same letter for a physical quantity, whether or not it has a dimension. Table I shows how the various quantities are nondimensionalized. Let us

TABLE I

Physical Quantity	Notation	Nondimensionalized by
Unperturbed velocity	U_0	$U_{0\infty}$
Perturbed velocity	u, v	$U_{0\infty}$
Position coordinates	x, x_0	L
Position coordinate	η	k^{-1}
Time	τ	$(U_{0\infty} k^{-1})^{-1}$
Composition variable	ξ, ξ_0	$C_p T_{0\infty}/A$

TABLE I (Continued)

Physical Quantity	Notation	Nondimensionalized by
Pressure	p, p_0	$p_{0\infty}$
Density	ρ, ρ_0	$\rho_{0\infty}$
Temperature	T	$T_{0\infty}$

introduce the quantity

$$\mathcal{L} = (kL)^{-1} \gg 1 \tag{4.2}$$

which is the ratio of flame thickness to the wave length of the disturbance. Neglecting terms of the order of M^2 in comparison with unity, we obtain the following system of equations: (cf., ref. 5)

Continuity Equation

$$\frac{d\rho_0}{dT}\left(u - \frac{1}{\mathcal{L}}\frac{\partial x}{\partial \tau}\right) + \rho_0\left(\frac{\partial u}{\partial T} + \frac{1}{\mathcal{L}}\frac{dx_0}{dT}\frac{\partial v}{\partial \eta}\right) = 0 \tag{4.3}$$

Momentum Equations

$$\left(\frac{dx_0}{dT}\right)^2\left[\frac{1}{\gamma M^2}\frac{\partial p}{\partial T} + \rho_0\left(u - \frac{1}{\mathcal{L}}\frac{\partial x}{\partial \tau}\right) + \frac{\partial u}{\partial T} + \frac{\rho_0}{\mathcal{L}}\frac{dx_0}{dT}\frac{\partial u}{\partial \tau}\right] = \frac{1}{Re}\left[\frac{4}{3}\frac{\partial^2 u}{\partial T^2}\frac{dx_0}{dT} + \right.$$
$$\left. -\frac{4}{3}\frac{\partial u}{\partial T}\frac{d^2x_0}{dT^2} + \frac{1}{3}\frac{1}{\mathcal{L}}\left(\frac{dx_0}{dT}\right)^2\frac{\partial^2 v}{\partial T\,\partial \eta} + \frac{1}{\mathcal{L}^2}\left(\frac{dx_0}{dT}\right)^3\frac{\partial^2 u}{\partial \eta^2} - \frac{1}{\mathcal{L}^2}\left(\frac{dx_0}{dT}\right)^2\frac{\partial^2 x}{\partial \eta^2} - \frac{4}{3}\frac{\partial^2 x}{\partial T^2}\right] \tag{4.4}$$

$$\left(\frac{dx_0}{dT}\right)^2\left[\frac{1}{\gamma M^2}\frac{1}{\mathcal{L}}\frac{dx_0}{dT}\frac{\partial p}{\partial \eta} + \frac{\partial \bar{v}}{\partial T} + \frac{\rho_0}{\mathcal{L}}\frac{\partial v}{\partial \tau}\frac{dx_0}{dT}\right] = \frac{1}{Re}\left[\frac{4}{3}\frac{1}{\mathcal{L}^2}\left(\frac{dx_0}{dT}\right)^3\frac{\partial^2 v}{\partial \eta^2} + \right.$$
$$\left. + \frac{1}{3}\frac{1}{\mathcal{L}}\frac{\partial^2 u}{\partial T\,\partial \eta}\left(\frac{dx_0}{dT}\right)^2 + \frac{\partial^2 \bar{v}}{\partial T^2}\frac{dx_0}{dT} - \frac{\partial \bar{v}}{\partial T}\frac{d^2x_0}{dT^2} - \frac{1}{3\mathcal{L}}\frac{\partial^2 x}{\partial T\,\partial \eta}\frac{dx_0}{dT}\right] \tag{4.5}$$

where

$$\bar{v} = v + \frac{1}{\mathcal{L}}\frac{\partial x}{\partial \eta} \tag{4.6}$$

Diffusion Equation

$$\left(\frac{dx_0}{dT}\right)^2\left[\beta\frac{1}{\mathcal{L}}\frac{\partial \xi}{\partial \tau}\frac{dx_0}{dT} + \rho_0\left(u - \frac{1}{\mathcal{L}}\frac{\partial x}{\partial \tau}\right) + \frac{\partial \xi}{\partial T}\right] + 2\frac{\partial x}{\partial T}\frac{dx_0}{dT} + \frac{1}{Re\,R}\left[-\frac{\partial^2 \xi}{\partial T^2}\frac{dx_0}{dT} + \right.$$
$$\left. + \frac{\partial^2 x}{\partial T^2} + \frac{d^2x_0}{dT^2}\frac{\partial \xi}{\partial T} + \frac{1}{\mathcal{L}^2}\frac{\partial^2 \xi}{\partial \eta^2}\left(\frac{dx_0}{dT}\right)^3 + \frac{1}{\mathcal{L}^2}\left(\frac{dx_0}{dT}\right)^2\frac{\partial^2 x}{\partial \eta^2}\right] = 0 \tag{4.7}$$

where Lewis number has been taken to be unity.

Energy Equation

$$\left(\frac{dx_0}{dT}\right)^2\left[\rho_0\left(u-\frac{1}{\mathcal{L}}\frac{\partial x}{\partial \tau}\right)\right]+2\frac{\partial x}{\partial T}\frac{dx_0}{dT}+\frac{1}{Re\,Pr}\left[\frac{\partial^2 x}{\partial T^2}+\frac{1}{\mathcal{L}^2}\left(\frac{dx_0}{dT}\right)^2\frac{\partial^2 x}{\partial \eta^2}\right]=0 \tag{4.8}$$

If we subtract (4.8) from (4.7), we get a second order homogeneous equation in ξ . The solution of this equation satisfying the boundary conditions $\xi = 0$ at both the upstream and the downstream infinity is obviously

$$\xi = 0 \tag{4.9}$$

so that, when Lewis number is taken as one, the temperature and composition variables play exactly the same role.

Notice that inside the flame terms containing $\frac{\partial x}{\partial T}$ and $\frac{\partial^2 x}{\partial T^2}$ can be omitted from all the equations. Outside the flame, $\frac{dx_0}{dT} \longrightarrow \infty$ so that as far as stability is concerned the energy equation will intervene only inside the flame where it assumes the form

$$\rho_0\left(u-\frac{1}{\mathcal{L}}\frac{\partial x}{\partial \tau}\right)+\frac{1}{\mathcal{L}^2 Re\,Pr}\frac{\partial^2 x}{\partial \eta^2}=0 \tag{4.10}$$

We thus see that conduction will influence the stability of flame if $Re\,Pr\,\mathcal{L}$ is of the order of unity. The mechanism of its action is also clear. In Landau's case (ref. 3) the assumption of a constant flame velocity is justified as $\mathcal{L} \gg 1$ in which case (4.10) becomes

$$u-\frac{1}{\mathcal{L}}\frac{\partial x}{\partial \tau}=0 \tag{4.11}$$

Eq. (4.11) shows that when $\mathcal{L}$ is not too large, conduction modifies the flame speed in such a way that the latter depends on the flame curvature. Thus we have a case similar to that considered by Markstein (ref. 4).

5. Complete Solution with Conduction In this calculation we shall neglect the effect of viscosity. We shall limit ourselves to the case:

$$Re \sim 1\ , \quad \mathcal{L} \gg 1\ , \quad Pr\,\mathcal{L} \sim 1 \tag{5.1}$$

Inside the flame, we have

$$\frac{d\rho_0}{dT}\left(u - \frac{1}{\alpha}\frac{\partial x}{\partial \tau}\right) + \rho_0\frac{\partial u}{\partial T} = 0 \tag{5.2}$$

$$\frac{1}{\gamma M^2}\frac{\partial p}{\partial \tau} + 2\frac{\partial u}{\partial T} = 0 \tag{5.3}$$

$$\frac{\partial v}{\partial T} = 0 \tag{5.4}$$

$$\rho_0\left(u - \frac{1}{\alpha}\frac{\partial x}{\partial \tau}\right) + \frac{1}{Re\,Pr\,\Omega^2}\frac{\partial^2 x}{\partial \eta^2} = 0 \tag{5.5}$$

This system of equations can also be obtained by an expansion procedure in a power series of Ω^{-1} . (See ref. 5) Since $\frac{\partial x}{\partial \tau} = 0$, we see that eq. (5.5) implies eq. (5.2), the latter, therefore, need not be considered further. If we assume that $x \sim De^{ik\eta+\omega t}$, eq. (5.5), applied to two sides of the flame, gives the two relations:

$$\begin{aligned} u_1 &= D\left(\omega + \frac{k^2\lambda}{C_p\rho_{0\infty}}\right) \\ u_2 &= D\left(\omega + \frac{\alpha k^2\lambda}{C_p\rho_{0\infty}}\right) \end{aligned} \tag{5.7}$$

where we have reverted to the use of quantities with dimensions. From (5.4)

$$iv_2 - iv_1 = U_{0\infty}Dk(\alpha - 1) \tag{5.8}$$

Finally from (5.3),

$$p_2 - p_1 = -2U_{0\infty}Dk^2\frac{\lambda}{C_p}(\alpha - 1) \tag{5.9}$$

Outside the flame the equations governing the disturbances are given by:

$$\begin{aligned} &\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \\ &\frac{\partial u_1}{\partial t} + U_{0\infty}\frac{\partial u_1}{\partial x} = -\frac{1}{\rho_{0\infty}}\frac{\partial p_1}{\partial x} \\ &\frac{\partial v_1}{\partial t} + U_{0\infty}\frac{\partial v_1}{\partial x} = -\frac{1}{\rho_{0\infty}}\frac{\partial p_1}{\partial y} \end{aligned} \tag{5.10}$$

in the region ahead of the flame front and by

$$
\begin{aligned}
&\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \\
&\frac{\partial u_2}{\partial t} + \alpha U_{0\infty}\frac{\partial u_2}{\partial x} = -\alpha \frac{1}{\rho_{0\infty}}\frac{\partial p_2}{\partial x} \\
&\frac{\partial v_2}{\partial t} + \alpha U_{0\infty}\frac{\partial v_2}{\partial x} = -\alpha \frac{1}{\rho_{0\infty}}\frac{\partial p_2}{\partial y}
\end{aligned}
\tag{5.11}
$$

in the region occupied by the burned gas. Assuming a solution which depends on y and t through the factor $e^{iky+\omega t}$ and that Re $\omega > 0$ (the unstable case), the solution can be written down without difficulties (see ref. 3). At x = o, these solutions yield the values:

$$
\left.
\begin{aligned}
&p_1 = A_1 \\
&p_2 = A_2 \\
&u_1 = -\frac{kA_1}{\rho_{0\infty} U_{0\infty} k(\Omega+1)} \\
&u_2 = B_2 + \frac{A_2\alpha}{\rho_{0\infty} U_{0\infty}(\Omega-\alpha)} \\
&i v_1 = \frac{kA_1}{\rho_{0\infty} U_{0\infty} k(\Omega+1)} \\
&i v_2 = \frac{B_2\Omega}{\alpha} + \frac{A_2\alpha}{\rho_{0\infty} U_{0\infty}(\Omega-\alpha)}
\end{aligned}
\right\}
\tag{5.12}
$$

where

$$
\Omega = \frac{\omega}{U_{0\infty} k}
\tag{5.13}
$$

We have to determine the four unknowns: A_1, A_2, B_2, D using the four homogeneous equations: (5.7), (5.8), (5.9). Since A_1, A_2, B_2, D cannot be zero simultaneously, the determinant of the coefficients must be zero. This provides us with an eigenequation for Ω. Simplifying the determinant, we obtain the following equation:

$$
\Omega^2(1+\alpha) + 2\Omega\alpha\left(\frac{1}{R_e P_r \mathcal{L}} + 1\right) + \alpha\left[\frac{2\alpha}{R_e P_r \mathcal{L}} - (\alpha-1)\right] = 0
\tag{5.14}
$$

If $\mathrm{Re}\,\mathrm{Pr}\,\mathcal{L} \to \infty$, (5.14) reduces to Landau's equation

$$\Omega^2(1+\alpha) + 2\Omega\alpha + \alpha(1-\alpha) = 0 \qquad (5.15)$$

which always admits a solution whose real part is positive (unstable). An examination of (5.14) shows that it will have two roots with negative real parts if

$$\mathrm{Re}\,\mathrm{Pr}\,\mathcal{L} \leqslant \frac{2\alpha}{\alpha-1} \qquad (5.16)$$

which gives us a stability criterion. The number $2\alpha/(\alpha-1)$ is close to 2 for large α. It is always bigger than 2. Note that (5.16) agrees with the statement we made regarding (3.8).

6. Extension to the case Represented by (3.10) As mentioned before, if we should limit the application of (5.18) to the case $\mathcal{L} \gg 1$. Re, Pr $\ll$ 1, the importance of the stability criterion would be mainly theoretical. We now attempt to extend the range of validity of (5.18) by reducing $\mathcal{L}$. The decrease of $\mathcal{L}$ will have two effects:

1) New terms will become important in the equations.

2) It will alter the locations where the boundary conditions are applied.

Of course, an intrinsic difficulty associated with a small $\mathcal{L}$ (e.g., $\mathcal{L} \sim 1$) is to know what happens to our assumption of a fluid continuum. Since $D \ll K^{-1}$, the displacement of the constant-temperature surfaces becomes smaller and smaller as $\mathcal{L} \to 1$. In such a case, it should have no effect at all on the flame so that the term $\frac{\partial x}{\partial T}$ inside the flame can again be neglected. As a consequence, the energy equation inside the flame is still given by (5.5). The equation (4.9) is, of course, still valid. We shall limit ourselves to the study of conduction effect while ignoring the viscous effect so that the viscous terms in the momentum equation need not be considered for the moment. As $\mathcal{L}$ decreases, terms of the form $\frac{1}{\mathcal{L}}\frac{dx_0}{dT}u, \ldots$ appear in the continuity

and momentum equations. Since these terms invariably contain the factor $\frac{dx_0}{dT}$ which is of the order of $1/\alpha$, these terms are actually of the order of $1/\alpha\mathcal{L}$ Now in an actual flame, α is of the order of 10 so that unless $\mathcal{L}$ becomes too small, these terms can again be neglected. It thus follows that the equations valid inside the flame remain the same as those given in eqs. (5.2) to (5.5). The equations governing the disturbance outside the flame are of course unchanged when viscous effect is suppressed.

Next we examine the effect of decreasing the quantity $\mathcal{L}$ on the locations where the boundary conditions are applied. The solution valid outside the flame contains terms of the form $e^{\pm kx}$ and $e^{-\frac{\Omega}{\alpha}kx}$. This solution is joined to the solution inside the flame at the two borders of the flame. Since x is of the order of L or D, all the exponentials will tend to 1 if $\mathcal{L} \gg 1$. As $\mathcal{L}$ is decreased, this simplification does not apply. Taking the origin of our coordinate system at the middle of the flame, the boundary conditions must now be applied at

$$x = \pm \frac{L}{2} + D e^{iky+\omega t} \qquad (6.1)$$

When this is substituted into the exponentials $e^{\pm kx}$ and $e^{-\frac{\Omega}{\alpha}kx}$, the result can be written as a product of two terms corresponding to the two terms in (6.1). The term containing D will, of course, tend to 1, since $D \ll K^{-1}$. The term containing L does not present any difficulties as it enters into the formulas as a coefficient. The fact that L is not well defined does not matter either, since the coefficients are finally eliminated and do not enter into the expression of the stability criterion. In conclusion, we see that the stability criterion (5.18) is valid for $\mathcal{L} \geqslant 1$.

7. <u>Stable Mode</u> It is interesting to calculate the form of the stable mode. Instead of assuming Re $(\Omega) > 0$, we take

$$R_e(\Omega) < 0 \qquad (7.1)$$

The solution of the system of equation outside the flame can be constructed as before. At the boundary, we have

$$p_1 = A_1 \tag{7.2}$$

$$p_2 = A_2 \tag{7.3}$$

$$u_1 = B_1 - \frac{A_1}{\rho_{0\infty} U_{0\infty}(\Omega+1)} \tag{7.4}$$

$$u_2 = \frac{A_2 \alpha}{\rho_{0\infty} U_{0\infty}(\Omega-\alpha)} \tag{7.5}$$

$$i v_1 = B_1 \Omega + \frac{A_1}{\rho_{0\infty} U_{0\infty}(\Omega+1)} \tag{7.6}$$

$$i v_2 = \frac{A_2 \alpha}{\rho_{0\infty} U_{0\infty}(\Omega-\alpha)} \tag{7.7}$$

Substituting these into (5.5) and the integrated form of (5.3), (5.4), we have a system of four equations, homogeneous in the variables A_1, A_2, B_1 and

$$B_1 - \frac{A_1}{\rho_{0\infty} U_{0\infty}(\Omega+1)} = D\left(\omega + \frac{k^2 \lambda}{c_p \rho_{0\infty}}\right) \tag{7.8}$$

$$\frac{A_2 \alpha}{\rho_{0\infty} U_{0\infty}(\Omega-\alpha)} = D\left(\omega + \frac{\alpha k^2 \lambda}{c_p \rho_{0\infty}}\right) \tag{7.9}$$

$$A_2 - A_1 = -2 U_{0\infty} D k^2 \lambda \frac{\alpha-1}{c_p} \tag{7.10}$$

$$\frac{A_2 \alpha}{\rho_{0\infty} U_{0\infty}(\Omega-\alpha)} - B_1 \Omega - \frac{A_1}{\rho_{0\infty} U_{0\infty}(\Omega+1)} = U_{0\infty} D k (\alpha-1) \tag{7.11}$$

Setting the determinant of the system zero, we have

$$\Omega^2(\alpha+1) + 2\Omega\alpha\left(\frac{1}{Re\,Pr\,\mathcal{L}} - 1\right) - \frac{2\alpha}{Re\,Pr\,\mathcal{L}} + \alpha(\alpha-1) = 0 \tag{7.12}$$

This equation shows that if

$$R_e P_r \mathcal{L} < 1 \tag{7.13}$$

there is at least one stable mode. To be more precise, there will be only one stable mode if $R_e P_r \mathcal{L} < \frac{2}{\alpha - 1}$ while two such modes are found for $R_e P_r \mathcal{L}$ greater than $\frac{2}{\alpha-1}$ but less than unity. Finally, if

$$1 < R_e P_r \mathcal{L} < \frac{2\alpha}{\alpha - 1} \tag{7.14}$$

the solution is still stable since there is no unstable solution for $R_e P_r \mathcal{L} < \frac{2\alpha}{\alpha-1}$. Here Ω is purely imaginary and the detailed structure of the disturbance at any instant can be calculated by solving an initial value problem.

8. Effect of Viscosity Here we assume the conduction effect can be neglected. We shall consider the case

$$R_e \mathcal{L} \sim 1 \tag{8.1}$$

Applying a similar reasoning as used in the previous sections, we find that the equations valid inside the flame are

$$u = \frac{1}{\mathcal{L}} \frac{\partial x}{\partial \tau} \tag{8.2}$$

$$\frac{1}{\gamma M^2} \frac{\partial p}{\partial T} = \frac{1}{R_e \mathcal{L}} \left[\frac{1}{3} \frac{\partial^2 v}{\partial T \partial \eta} - \frac{1}{\mathcal{L}} \frac{\partial^2 x}{\partial \eta^2} \right] \tag{8.3}$$

$$\frac{\partial \bar{v}}{\partial T} = 0 \tag{8.4}$$

where $\bar{v}$ is given by (4.6). Substituting (4.6) into (8.3) and making use of (8.4), we obtain:

$$\frac{1}{\gamma M^2} \frac{\partial p}{\partial T} = \frac{4}{3} \frac{1}{R_e \mathcal{L}^2} \frac{\partial^2 x}{\partial \eta^2} \tag{8.5}$$

which may be used instead of (8.3).

Outside the flame we must use the linearized form of Navier-Stokes equations. Thus, instead of (5.10) and (5.11), we have:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8.6}$$

$$\rho_0 \frac{\partial u}{\partial t} + \rho_0 U_0 \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{8.7}$$

$$\rho_0 \frac{\partial v}{\partial t} + \rho_0 U_0 \frac{\partial v}{\partial x} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{8.8}$$

where ρ_0, U_0 stand for $\rho_{0\infty}$, $U_{0\infty}$ in the region ahead of the flame, while they stand for $\rho_{0\infty}/\alpha$, $U_{0\infty}\alpha$ in the region behind the flame. The appropriate solutions, applicable to the upstream and downstream regions, of the above system can be easily constructed. To obtain a stability criterion, we need only the values of the various flow variables at the boundaries of the flame. At the border adjoining the region occupied by the unburned gas,

$$\begin{aligned} p_1 &= A_1 \\ u_1 &= -A_1/\rho_{0\infty} U_{0\infty}(\Omega+1) \\ iv_1 &= A_1/\rho_{0\infty} U_{0\infty}(\Omega+1) \end{aligned} \tag{8.9}$$

At the border adjoining the region occupied by the burned gas,

$$\begin{aligned} p_2 &= A_2 \\ u_2 &= B_2 + A_2\alpha/\rho_{0\infty} U_{0\infty}(\Omega-\alpha) \\ iv_2 &= -B_2\lambda_2 + A_2\alpha/\rho_{0\infty} U_{0\infty}(\Omega-\alpha) \end{aligned} \tag{8.10}$$

where λ_2 is approximately given by

$$\lambda_2 = -\frac{\Omega}{\alpha} - \frac{1}{\mathcal{L} Re} \tag{8.11}$$

Now from (8.2), (8.4) and (8.5), we have

$$\begin{aligned} u_2 &= u_1 = D\omega \\ iv_2 - iv_1 &= U_{0\infty} Dk(\alpha-1) \\ p_2 - p_1 &= \frac{4}{3}\frac{1}{Re\,\mathcal{L}}(\alpha-1)\rho_{0\infty} U_{0\infty}^2 Dk \end{aligned} \tag{8.12}$$

Substituting (8.9), (8.10), (8.11) into (8.12), we obtain a system of homogeneous equations in A_1, A_2, B_2 and D. Setting its determinant equal to zero,

we have

$$\alpha\Omega^2\left(1-\frac{1}{\mathcal{L}R_e}\right)-\alpha^2\left(1+\frac{4}{3\mathcal{L}R_e}\right)\sim 0 \tag{8.13}$$

One of the solution is

$$\Omega=\sqrt{\alpha\frac{1+\frac{4}{3\mathcal{L}R_e}}{1-\frac{1}{\mathcal{L}R_e}}} \tag{8.14}$$

as compared with $\Omega=\sqrt{\alpha}$ in Landau's case. Hence, a necessary condition for stability is

$$R_e<\frac{1}{\mathcal{L}} \tag{8.15}$$

A comparison of (5.16) and (8.15) shows that the flame is primarily stabilized by the influence of heat conduction rather than the effect of viscosity.

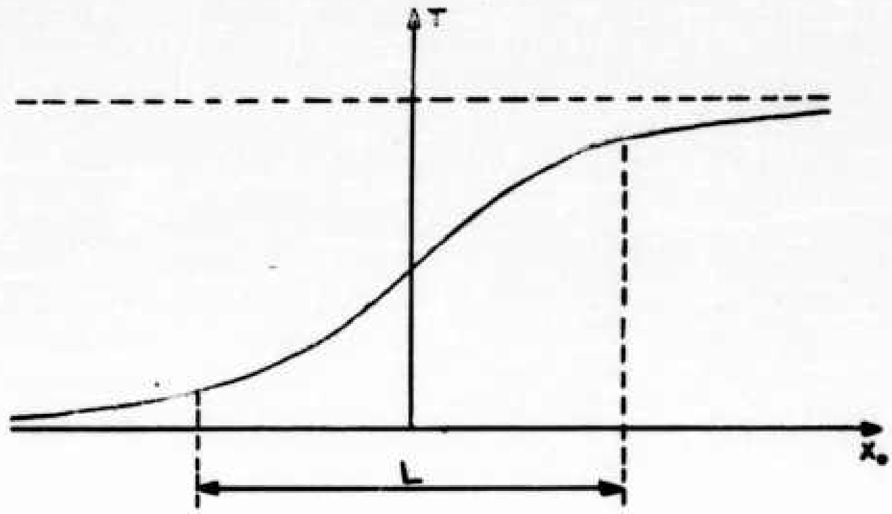

Figure 1

References

1. Hirschfelder, J. O., Curtis, C. F., Bird, R. B.: Molecular Theory of Gases and Liquids, Wiley & Son, N. Y., 1954

2. Penner, S.: Chemistry Problem in Jet Propulsion, Perg. Press, N. Y., 1958

3. Landau, L.: Acta physicochim (U.R.S.S.) 19, 77 (1944)

4. Markstein, G.: Fourth Symp. on Combustion, p.44, Williams and Wilkins, Baltimore, 1952.

5. Chu, B. T., Parlange, J. Y.: On the Stability of Laminar Flame, Brown University Rpt. Contract No. AF 49 (638)-646

UNCLASSIFIED

UNCLASSIFIED